NATIONAL ACADEMY OF SCIENCES OF UKRAINE

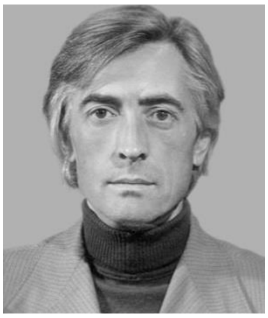
INSTITUTE OF APPLIED MATHEMATICS AND MECHANICS

WORKSHOP

"*CONTEMPORARY ANALYSIS AND NONLINEAR BOUNDARY PROBLEMS*"

Sloviansk, October 17-18, 2018

Information on the workshop: The workshop "*Contemporary Analysis and Nonlinear Boundary Problems*" is dedicated to the 80th anniversary of B.V. Bazaliy (1938-2012) and to the centennial anniversary of the National Academy of Sciences of Ukraine.



Bazaliy Borys Vasyliovych was a Ukrainian mathematician, Professor at the Institute of Applied Mathematics and Mechanics of NASU. His most influential contributions are in the area of free boundary problems, nonlinear PDE, and their applications to Mathematical Biology.

The workshop aims to review and discuss the latest trends in Free Boundary Problems, PDE and Analysis.

Organizing Committee:

Yaroslaw Bazaliy (USA), Yevgeniia Yevgenieva (Ukraine), Mykola Krasnoschok (Ukraine), Vasylyeva Nataliya (Ukraine)

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ABSTRACTS

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Harnack's inequality for double-phase parabolic equations

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This talk is related to parabolic equations with non standard growth conditions. Let Ω be a domain in $\mathbb{R}^n$, $T > 0$, $\Omega_T = \Omega \times (0, T)$. We study solutions to equation

$$u_t - \mathrm{div} A(x, t, \nabla u) = 0, \ (x, t) \in \Omega_T. \tag{1}$$

Let the functions $A : \Omega \times \mathbb{R}^1_+ \times \mathbb{R}^n \to \mathbb{R}^n$ meet the requirements:

- $A(\cdot, \cdot, \xi)$ are Lebesgue measurable for all $\xi \in \mathbb{R}^n$;
- $A(x, t, \cdot)$ are continuous for almost all $(x, t) \in \Omega_T$.

We also assume that the following structure conditions hold

$$A(x, t, \xi)\xi \geq \mu_1 G_a(|\xi|), \qquad |A(x, t, \xi)| \leq \mu_2 g_a(|\xi|), \tag{2}$$

where $g_a(|\xi|) := |\xi|^{p-1} + a(x, t)|\xi|^{q-1}$, $G_a(|\xi|) := g_a(|\xi|)|\xi|$, μ_1, μ_2 are positive constants;

$a(x, t) \geq 0$, $a(x, t) \in C^{\alpha, \frac{\alpha}{p_-}}(\Omega_T)$ with some positive $\alpha \in (0, 1]$, $p_- = \min(p, 2)$;

p, q satisfy the inequalities $\frac{2n}{n+1} < p < q \leq p + \alpha$.

In addition, we assume that $(A(x, t, \xi) - A(x, t, \eta))(\xi - \eta) > 0, \ \xi \neq \eta$.

We obtain the local properties of bounded solutions to equation (1): Hölder continuity of the solutions and Harnack's type inequality. These properties are basically characterized by the different types of degenerate behavior, according to the size of the coefficient $a(x, t)$ that determines the "phase". Indeed, on the set $\{a(x, t) = 0\}$ equation (1) has the growth of order p with respect to the gradient (this is so called "p-phase"). At the same time, this growth is of order q, if $a(x, t) > 0$ (this corresponds to $(p, q)-$ phase).

Yu.S. GORBAN

On weak solutions to nonlinear elliptic degenerate anisotropic equations with L^1-data

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We consider the Dirichlet problem

$$-\sum_{i=1}^{n}\Big(\nu_i(x)|u_{x_i}|^{q_i-2}u_{x_i}\Big)_{x_i} = F(x,u) \ \text{ in } \ \Omega, \ \ u=0 \ \text{ on } \partial\Omega, \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, $n \geqslant 2$, $q_i \in (1,n)$, $\nu_i : \Omega \to \mathbb{R}$, $\nu_i > 0$ a.e. in Ω, $\nu_i \in L^1_{\text{loc}}(\Omega)$, ${\nu_i}^{-1/(q_i-1)} \in L^1(\Omega)$, $F : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function.

Definition. *The function $u \in \overset{\circ}{W}{}^{1,1}(\Omega)$ is a weak solution of problem (1) if the following conditions hold:*

i) $F(x,u) \in L^1(\Omega)$;

ii) $\nu_i|u_{x_i}|^{q_i-2}u_{x_i} \in L^1(\Omega)$;

iii) *for every function $w \in C^1_0(\Omega)$ the equality holds*

$$\int_\Omega \left\{\sum_{i=1}^{n}\nu_i|u_{x_i}|^{q_i-2}u_{x_i}w_{x_i}\right\}dx = \int_\Omega F(x,u)w\,dx.$$

Let
$\overline{q} = n(\sum_{i=1}^{n} 1/q_i)^{-1}$,
and $p_m = n(\sum_{i=1}^{n}(1+m_i)/(m_iq_i) - 1)^{-1}$ for every $m \in \mathbb{R}^n$, $m_i > 0$.

Theorem. *Let the following conditions hold*

(i) $F(x,\cdot)$ *is nonincreasing on* $\mathbb{R}$ *for a.e.* $x \in \Omega$,

(ii) $F(\cdot,s) \in L^1(\Omega)$ *for any* $s \in \mathbb{R}$,

(iii) $\exists\, m,\sigma \in \mathbb{R}^n$, $m_i > 0$, $\sigma_i > 0 :$ $\overline{q}/(p_m(\overline{q}-1)) < q_i - 1 - 1/m_i$,

$$1/\nu_i \in L^{m_i}(\Omega);\ 1/\sigma_i < 1 - ((q_i-1)\overline{q})/(p_m(\overline{q}-1)),\ \nu_i \in L^{\sigma_i}(\Omega).$$

Then there exists a weak solution of problem (1).

This result continues researches in [1, 2, 3].

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[3] Yu.S. Gorban, *Existence of entropy solutions for nonlinear elliptic degenerate anisotropic equations,* Open Mathematics, **15**, (2017), 768–786.

Ye.Ya. KHRUSLOV

Homogenized models of the dynamics of suspensions

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We consider the system of equations which describes the motion of a mixture of small solid particles with the viscous incompressible liquid (suspension). We study the asymptotic behavior of the system solutions when the particles sizes vanish, and their number increases.

Depending on parameters of the mixture two asymptotic regimes of the suspension motion can be realized: the regime of the frozen-in particles or the regime of the filterable particles. We obtain the homogenized equations describing these regimes and study their solvability.

A.N. KOCHUBEI

Parabolic equations with p-adic spatial variables

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In this survey talk, we consider linear and nonlinear evolution equations for complex-valued functions of a real positive time variable and p-adic spatial variables. In the linear case, there is a well-developed theory of the class of p-adic parabolic equations having both common and different features compared with the classical theory of parabolic equations. In the nonlinear case, we deal with a non-Archimedean counterpart of the fractional porous medium equation. Developing, as a tool, an L^1-theory of Vladimirov's p-adic fractional differentiation operator, we prove m-accretivity of the appropriate nonlinear operator, thus obtaining the existence and uniqueness of a mild solution.

M. KRASNOSHCHOK

Fractional diffusion in domains with fixed and free boundaries

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Let Q be a bounded domain in R^N with smooth boundary. First we consider time-fractional equation in $Q \times (0,T)$

$$\partial^{\alpha}_{0+,t}u(x,t) - a_{ij}(x,t)u_{x_ix_j}(x,t) + a_i(x,t)u_{x_i}(x,t) + a_0(x,t)u(x,t) = f(x,t),$$

supplemented with the initial condition

$$u(x,0) = \varphi(x) \quad \text{in}\, Q,$$

and prove classical solvability to initial-boundary problems with the boundary conditions of the following types:

$$u = \psi_1(x,t),$$

$$(b(x,t)\cdot\nabla u(x,t)) + b_0(x,t)u(x,t) = \psi_2(x,t),$$

$$\partial^{\alpha}_{0+,t}u(x,t) + (b(x,t)\cdot\nabla u(x,t)) + b_0(x,t)u(x,t) = \psi_3(x,t).$$

Here,

$$\partial^{\alpha}_{\rho+,t}u(x,t) = \frac{1}{\Gamma(1-\alpha)}\left[\frac{\partial}{\partial t}\int_{\rho}^{t}(t-\tau)^{-\alpha}(u(x,\tau)-u(x,\rho))d\tau\right]$$

is a regularized fractional derivative of the order $\alpha \in (0,1)$.

Then we investigate the one-dimensional Stefan problem and find the unknown function $u(y,t)$ and the free boundary $s(t)$ by the following conditions:

$$\partial^{\alpha}_{l(x)+,t}u(y,t) = u_{yy}(y,t),\ (x,t) \in Q^s_{\sigma} = \{(y,t)|\ y \in (0,s(t)),\ t \in [0,\sigma)\},$$

$$s(0) = s_0,$$

$$u(y,0) = \phi(y) > 0,\ \ x \in (0,s_0)$$

$$u(0,t) = f(t), \quad t \in [0,\sigma),$$

$$u(t,s(t)) = 0, \quad t \in [0,\sigma),$$

$$\partial^{\alpha}_{0+,t}s = -u_y(s(t),t), \quad t \in (0,\sigma).$$

Here $l(y)$ satisfies conditions

$$l(y) = 0, y \in [0,s_0],\ \ s(l(y)) = y,\ \ y > s_0.$$

We reduce this problem to a nonlinear integral equation to establish the solvability.

A.I. MARKOVSKII

On identifying the formation pressure and filtration coefficients of two gas-bearing formations

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During gas field exploitation, there are cases when a well opens several gas-bearing formations simultaneously. Moreover, there may occur gas cross-flows between the formations, which is considered to be economically undesirable. Given that a wellhead pressure p is the only parameter by changing which it is possible to control the production rate, it is important to know how production rates of exploited formations functionally depend on such a pressure. The pressure problem on two formations has already been considered in the author's previous studies. Thus, it is necessary to know the formation pressures and filtration coefficients. However, this condition doesn't often hold in practice. This study considers the problem of determining unknown formation pressures and filtration coefficients of two gas-bearing formations opened by a single well on the basis of stationary wellhead pressure conditions and the total production rate measurements data. The problem is reduced to solving a complicated system of three nonlinear equations. The research involves an algorithm and an example of a numerical solution, with software processing of the one of possible options.

T.A. MEL'NYK

Asymptotic analysis of semilinear parabolic problems in thin star-shaped junctions

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We consider semilinear parabolic problems with Robin's nonlinear perturbed boundary conditions in thin star-domains, consisting of several thin curvilinear cylinders, which are connected through a region (node) of a small diameter. The asymptotic behavior of solutions to these problems will be analyzed, when a thin star-shaped junction is converted into a graph. Depending on the parameters in the Robin conditions, we establish different asymptotic behavior of solutions. In each case, the boundary value problem

is obtained on the graph with the corresponding Kirchhoff conditions at the vertex (in some cases they are non-standard).

V.S. OVERKO

The influence of Non-Newtonian effects on features of blood flow in the left ventricle

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The left ventricular torsion motion plays an important role for LV ejection and forming the complex structure of flow in the aorta. We study two models: N-model (blood is assumed as a Newtonian fluid) and NN-model (blood is assumed as a Non-Newtonian fluid). The results for the N-model demonstrate that the torsional motion influences to the pattern of flow in the left ventricle. That creates the condition for faster filling of the left ventricle because the gradient of the pressure is a significantly greater. The disturbance of torsional motion leads to deterioration of blood circulation and correspondingly decreasing oxigenation. The patterns of flow are very interesting for the cross section of the aorta. In the initial part of the systole, the flow pattern is more homogeneous for the NN-model than for the N-model. The middle part of the systole does not show significant differences both in the N-model and in the NN-model. It should be remarked that Non-Newton properties of blood have significant effect on the development of secondary flow, especially if the flow has the slowdown. The decreasing of velocity's magnitude leads to disappear the zones with high shear stresses. It can be useful, on our opinion, for the more precision diagnosis.

E.A. SEVOST'YANOV, A.A. MARKYSH

On Sokhotski–Casoratti-Weierstrass theorem on metric spaces

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Let (X, d, μ) and (X', d', μ') be metric spaces with metrics d and d' and locally finite Borel measures μ and μ', correspondingly. Let us consider condition $\mathbf{A}$: for all $\beta : [a,\, b) \to X'$ and $x \in f^{-1}\left(\beta(a)\right)$, a mapping $f : D \to X'$ has a maximal f-lifting in D starting at x.

We say that a function $h : \overline{X} \times \overline{X} \to \mathbb{R}$ meets the requirement **B** on $\overline{X} := X \cup \infty$, if the following conditions hold:
$\mathbf{B}_1$: h is a metric on $\overline{X}$;
$\mathbf{B}_2$: $(\overline{X}, h)$ is a compact metric space;
$\mathbf{B}_3$: $h(x, y) \leqslant d(x, y)$ for every $x, y \in X$.

A mapping $f : G \setminus \{x_0\} \to G'$ is a *ring Q-mapping at a point $x_0 \in \partial G$ with respect to (p, q)-moduli*, if the inequality $M_p(f(\Gamma(S_1, S_2, A))) \leqslant \int\limits_{A \cap G} Q(x)\eta^q(d(x, x_0))\, d\mu(x)$ holds for each ring $A = A(x_0, r_1, r_2) = \{x \in X : r_1 < d(x, x_0) < r_2\}$, $0 < r_1 < r_2 < \infty$ and every measurable function $\eta : (r_1, r_2) \to [0, \infty]$ with $\int\limits_{r_1}^{r_2} \eta(r)dr \geqslant 1$.

Theorem. *Let $2 \leqslant \alpha, \alpha' < \infty$, $1 \leqslant q \leqslant \alpha$, $\alpha' - 1 < p \leqslant \alpha'$ and let (X, d, μ) be locally compact Ahlfors α-regular metric space. Let (X', d', μ') be Ahlfors α'-regular proper path connected, locally connected metric space where $(1; p)$-Poincaré inequality holds. Let $G := D \setminus \{\zeta_0\}$ be a domain in X, which is locally path connected at $\zeta_0 \in D$. Assume that $Q \in FMO(\zeta_0)$, and there exists a function h satisfying conditions **B**.*

*If an open discrete ring Q-mapping $f : D \setminus \{\zeta_0\} \to X$ at ζ_0 with respect to (p, q)-moduli satisfies the condition **A** and ζ_0 is an essential singularity of f, then $f(V \setminus \{\zeta_0\})$ is dense in X' for an arbitrary neighborhood V of ζ_0.*

M.A. SHAN

Removable singularities for anisotropic parabolic equations

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This paper is devoted to obtaining conditions for a removable singularity at a point for solutions of quasilinear parabolic equations:

$$u_t - \sum_{i=1}^{n} \left(u^{m_i - 1} u_{x_i}\right)_{x_i} = 0, \tag{1}$$

$$u_t - \sum_{i=1}^{n} \left(u^{m_i - 1} u_{x_i}\right)_{x_i} + f(u) = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} - \sum_{i=1}^{n} \left(u^{m_i - 1} u_{x_i}\right)_{x_i} + \sum_{i=1}^{n} |u_{x_i}|^{q_i} = 0, \tag{3}$$

We focus on the solutions satisfying the initial condition

$$u(x,0) = 0, \;\; x \in \Omega \setminus \{0\} \tag{4}$$

where Ω is a bounded domain in R^n, $n \geq 2, t \in (0,T), 0 < T < +\infty$, $0 \in \Omega$.

Let the exponents m_i, q_i $i = \overline{1,n}$ satisfy the following conditions

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq \ldots \leq m_n < m + \frac{2}{n}, m = \frac{1}{n}\sum_{i=1}^{n} m_i,$$

$$\frac{2+nm}{1+n} \leq q < 2, \quad \max_{0 \leq i \leq n} q_i < q\left(1+\frac{1}{n}\right), \quad \frac{1}{q} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{q_i}.$$

The main difficulty deals with some m_i is less than 1 (singular case), and another part of m_i is a greater than 1 (degenerate case). We find the universal approach to study the properties of solutions to the anisotropic porous medium equation which is independent of the anisotropic exponents m_i. We establish the pointwise condition on removable singularity of solutions for equation (1) [1]. We also obtain the pointwise estimates of solutions depending on the relations between the exponents m_i and q_i (for equation (3) [3]), m_i and q (for equation (2) in the case $f(u) = u^q$ [2]). That guarantees the removability of the point singularity. The proof of the removability is based on new a priori estimates of "wide" type solutions. In particular, we obtain the Keller-Osserman type estimate of the solution to problems (2), (4) and (3), (4).

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D.G. SHEPELSKY

The focusing NLS equation with step-like oscillatory backgrounds: scenarios of long-time asymptotics

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We consider the Cauchy problem for the focusing nonlinear Schrödinger (NLS) equation $iq_t + q_{xx} + 2|q|^2q = 0$, $x \in \mathbb{R}$, $t \geq 0$, with the initial data $q(x,0) = q_0(x)$ approaching, for lagre $|x|$, different oscillatory backgrounds:

$$q_0(x) \sim \begin{cases} A_1 e^{-2iB_1x+i\phi_1}, & x \to -\infty \\ A_2 e^{-2iB_2x+i\phi_2}, & x \to +\infty, \end{cases}$$

where $\{A_j, B_j, \phi_j\}_1^2$ are real constants such that $A_j > 0$. The solution $q(x,t)$ is assumed to approach the associated plane wave backgrounds for all $t \geq 0$:

$$q(x,t) = q_{0j}(x,t) + o(1) \quad \text{as } x \to (-1)^j\infty,$$

where

$$q_{0j}(x,t) = A_j e^{-2iB_jx+2i\omega_jt+i\phi_j} \quad \text{with } \omega_j = A_j^2 - 2B_j^2, \quad j = 1,2.$$

Our main objective is the study of the long-time behavior of the solution to the Cauchy problem above.

The tool that we use is the Riemann-Hilbert (RH) formulation of the problem, which can be viewed as a version of the inverse scattering transform (IST) method, and the subsequent large-t asymptotic analysis of the RH problem. This method is well-developed for problems with "zero boundary conditions", where the solution is assumed to decay to 0 as $x \to \pm\infty$ for all $t \geq 0$.

In this talk, we discuss the case $B_1 \neq B_2$, $A_j \neq 0$. We present several possible scenarios (depending on the ratios $\frac{A_j}{B_j}$) of the long-time behavior. These scenarios involve asymptotic formulas defined in terms of objects related to compact Riemann surfaces of different genus, varying from 0 to 3.

I.I. SKRYPNIK [1], K.O. BURYACHENKO [2]

Riesz potentials and pointwise estimates of solutions to anisotropic porous medium equation

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In this talk we consider anisotropic porous medium equations

$$u_t - \sum_{i=1}^{n} \left(u^{m_i-1} u_{x_i}\right)_{x_i} = f, \ \text{in} \ \Omega_T, \, u \geq 0,$$

$$1 - \frac{2}{n} < m_1 \leq m_2 \leq ... \leq m_{n-1} < m_n < m + \frac{2}{n}, \, m = \frac{1}{n} \sum_{i=1}^{n} m_i,$$

where $\Omega_T = \Omega \times (0, T)$, Ω is a bounded domain in $\mathbb{R}^n$, $0 < T < \infty$, $n \geq 2$, $f \in L^1_{loc}(\Omega_T)$.

We consider the case when some m_i can be less than 1 (so called "singular" case) and the other m_i can be greater than 1 (so called "degenerate" case). Our aim is to establish basic qualitative properties such as local boundedness and continuity of weak solutions in terms of a linear anisotropic Riesz potential of the right-hand side function f.

Due to introducing $m^+ = \max(m_n, 1)$, proofs of our results are independent of the cases $m_n > 1$ or $m_n < 1$ (see, e.g., [1, 2, 3] and [5]). Moreover, we can observe here that the Riesz potential plays the same role in the linear statement, although this fact is already well known, see [6].
For details of announced results, we refer the interested reader to paper [4].

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R.M. TARANETS

The spreading of surfactant on thin films

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We study the dynamics of thin liquid films influenced by an insoluble surfactant (a surface tension reducing agent) on a horizontal plane in the presence of gravity. The motion of the film is modelled in the lubrication approximation by a fourth order coupled system of nonlinear degenerate partial differential equations (see [1]). We prove the existence of nonnegative weak solutions to the system with nonnegative initial data (see [2]). Also, we find power-law behaviour for finite speed propagation and define sufficient conditions for waiting time phenomena (see [3]).

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N. VASYLYEVA

Moving boundary problems governed by anomalous diffusion in multidimensional domains

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Fractional partial differential equations (FPDE) play a key role in the description of the so-called anomalous phenomena in nature and in the theory of complex systems (see e.g. [1]). In particular, these equations provide a more faithful representation of the long-memory and nonlocal dependence of many anomalous processes. In the last two decades, FPDE have drawn an increasing attention in several research fields, including Mathematical Modeling, Electromagnetism, Polymer Science, Hydrology, Geophysics, Biophysics, Finance and Viscoelasticity.

Here we focus on the anomalous diffusion version of the one- and two-phase quasistationary Stefan problems (the fractional Hele-Shaw problem and the fractional Muskat problem) in the multidimensional case. In the case of zero surface tension of the moving boundary, the one-phase fractional Hele-Shaw problem is a mathematical model of a solute drug release from a polymer matrix. The relevant mathematical models of drug release from a polymeric matrix are powerful tools in studies of controlled-release drug system.

We assert the one-valued classical solvability of the "fractional" Hele-Shaw and Muskat problems locally in time. In the two-dimensional case we construct numerical solutions to the one-phase Hele-Shaw problem in subdiffusion case.

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M.V. VOITOVYCH

Local boundedness of solutions of some high-order quasilinear elliptic equations in limit cases

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Let $m, n \in \mathbb{N}$, $m \geqslant 3$ and $n > 2(m-1)$. Let Ω be a bounded open domain in $\mathbb{R}^n$. We consider the $2m$-order quasilinear partial differential equation

$$\sum_{1\leqslant|\alpha|\leqslant m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, \nabla_m u) = f(x),\ x \in \Omega,$$

where $f : \Omega \to \mathbb{R}$, $\nabla_m u = \{D^\alpha u : 1 \leqslant |\alpha| \leqslant m\}$.

Let the coefficients A_α $(1 \leqslant |\alpha| \leqslant m)$ be Carathéodory functions satisfying certain growth and coercivity conditions (see, e.g., [1, 2]), suitable for the energy space $W^{1,q}(\Omega) \cap W^{m,p}(\Omega)$, where $2n(m-2)/[n(m-1)-2] < p < n/m$, $\max(\overline{p}, mp) < q \leqslant n$, $\overline{p} = 2p/[p(m-1) - 2(m-2)]$.

Here, we discuss how the local boundedness of solutions to the equation depends on the integrability of the right-hand side f. We consider the following cases:
(i) $n > q$ and $f \in L^{n/q,\, 1/(q-1)}(\Omega)$;
(ii) $n = q$ and

$$f \in L(\log L)^{n-1}(\log\log L)^{n-2} \cdots (\log \cdots \log L)^{n-2}(\log \cdots \log L)^{n-2+\epsilon}(\Omega)$$

with some $\epsilon > 0$.

Similar questions to the second-order equations are studied in [3, 4].

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Large solutions of quasilinear parabolic equations of diffusion – nonlinear degenerate absorption type

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Let $\Omega \subset R^n$ be a bounded domain with a smooth boundary $\partial\Omega \in C^2$. We analyze the following problem for a quasilinear parabolic equations of diffusion – nonlinear degenerate absorption type in the cylindrical domain $Q = (0, T) \times \Omega,\ 0 < T < \infty$:

$$(|u|^{p-1}u)_t - \Delta_p(u) = -b(t,x)|u|^{\lambda-1}u \quad \lambda > p > 0,$$

$$u = \infty \ \text{ on } (0,T) \times \partial\Omega,\ u = \infty \ \text{ on } \{0\} \times \Omega,$$

We discuss the asymptotic properties of weak (energy) solutions to the problem [1].

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